香港培正中學第一屆數學邀請賽

Pui Ching Middle School 1st Invitational Mathematics Competition

個人賽（中一組）

Individual Event (Secondary 1)

時限：1 小時

Time allowed: 1 hour

參賽者須知：

Instructions to Contestants:

1. **本卷共設** 20 **題，總分為** 100 **分。**

   There are 20 questions in this paper and the total score is 100.

2. **除特別指明外，本卷內的所有數均為十進制。**

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. **除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。**

   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. **所有答案填在答題紙指定的空位上。毋須呈交計算步驟。**

   Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. **不得使用計算機。**

   The use of calculators is not allowed.

6. **本卷的附圖不一定依比例繪成。**

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

Questions 1 to 5 each carries 2 marks.

1. 100000000001 **除以** 3 **時，餘數是多少？**

   What is the remainder when 100000000001 is divided by 3?

2. **求一個質數** p**，使得** $23p+1$ **也是質數。**

   Find a prime number p such that $23p+1$ is also prime.

3. **小明和小芬各自想了一個數，已知小明想到的數比小芬的大。若兩數之和為** 2002**，兩數之差為** 1001**，則小芬想到的數是多少？**

   Michael and Fanny each thought of a number. It is known that Michael' s number is larger than Fanny' s. If the sum of the two numbers is 2002 and their difference is 1001, what is the number Fanny thought of?

4. **在某餐廳中，午市套餐包括餐湯、主菜及飲品。顧客可從** 3 **款餐湯、**6 **款主菜及** 12 **款飲品中各選一款。問午市套餐共有多少個不同的組合？**

   In a canteen, a set lunch consists of a soup, a main dish and a drink. Customers could choose one among each of the 3 choices of soups, 6 choices of main dishes and 12 choices of drinks. How many different combinations of the set lunch are there?

5. **設** N **為正整數。當** 2002 **減去** N **後，所得的數可同時被** 2**、**3**、**4**、**5**、**6 **整除。求** N **的最小可能值。**

   Let N be a positive integer. When N is subtracted from 2002, the resulting number is divisible by 2, 3, 4, 5 and 6. Find the smallest possible value of N.

第 6 至第 10 題，每題 4 分。

Questions 6 to 10 each carries 4 marks.

6. **2002 人參加一個淘汰賽。在每個回合中，參賽者分成二人一組對壘，勝方可晉級下一回合，負方則被淘汰出局。若某回合中參賽者的數目為奇數，則抽出一人自動晉身下一回合。比賽一直繼續，直至剩下一名參賽者為冠軍。問共要進行多少場比賽？**

   2002 people have joined a tournament, which adopts a knock-out system. In each round, the players are grouped into pairs to compete: the winner can proceed to the next round while the loser is knocked out. If the number of people in a certain round is odd, one person, chosen randomly, can proceed to the next round without any match. The tournament continues until only one person is left, who is then the champion. How many matches will be played altogether?

7. **求** $7\times9\times11\times\cdots\times1999\times2001\times2003$ **的個位數字。**

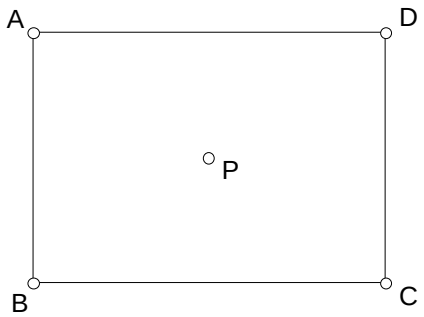
   Find the unit digit of $7\times9\times11\times\cdots\times1999\times2001\times2003$.

8. **某年，X 國舉辦第二屆數學邀請賽。男參賽者的數目比第一屆增加了 20%，女參賽者的數目則比第一屆增加了 80%。若第二屆的參賽者總數比第一屆多 32%，則第二屆女參賽者的數目佔全部參賽者的幾分之幾？**

   In a certain year, Country X organised the second invitational mathematics competition. The number of male contestants was increased by 20% compared to the first competition while the number of female contestants was increased by 80% compared to the first competition. If total number of contestants in the second competition was 32% more than that in the first competition, what fraction of the total number of contestants was the number of female contestants in the second competition?

9. **圖中，*P* 為長方形 *ABCD* 的中心。若 *P* 到 *AB* 的距離為 *P* 到 *BC* 的距離之兩倍，且 *ABCD* 的周界為 120，求 *ABCD* 的面積。**

   In the figure, *P* is the centre of rectangle *ABCD*. If the distance from *P* to *AB* is twice the distance from *P* to *BC*, and the perimeter of *ABCD* is 120, find the area of *ABCD*.



10. **一個三角形被放在一個邊長為 10 的正方形內。三角形的最大可能面積是多少？**

    A triangle is put inside a square of side length 10. What is the largest possible area of the triangle?

第 11 至第 15 題，每題 6 分。

Questions 11 to 15 each carries 6 marks.

11. **設 x 為正整數。若 2002 和 x 的最小公倍數為 30030，問 x 有多少個不同的可能值？**

Let x be a positive integer. If the lowest common multiple (L.C.M.) of 2002 and x is 30030, how many different possible values of x are there?

12. **陳先生有兩名兒子，他們的年齡相差一歲。在 1997 年，陳先生的年齡是兩名兒子年齡之和的八倍。在 2002 年，陳先生的年齡是兩名兒子年齡之和的三倍。當陳先生的長子出生時，陳先生的年齡是多少？**

Mr Chan has two sons whose ages differ by 1. In 1997, the age of Mr Chan was 8 times the sum of his sons'. In 2002, the age of Mr Chan was 3 times the sum of his sons'. How old was Mr Chan when his elder son was born?

13. **對於正整數 n，若 n^2-2 為 7 的倍數，則 n 稱為「好數」。在首 2002 個正整數中，有多少個是「好數」？**

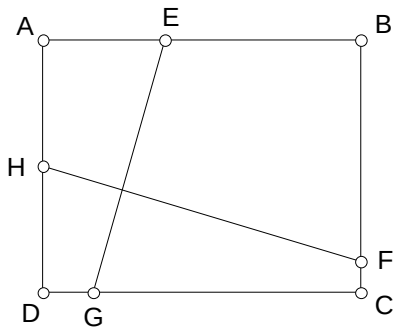
For a positive integer n, we say that n is 'good' if n^2-2 is a multiple of 7. How many of the first 2002 positive integers are 'good'?

14. **求最小的質數 p，使得 $2002-p$ 和 $2002+p$ 均為質數。**

Find the smallest prime number p for which both $2002-p$ and $2002+p$ are prime.

15. **如圖所示，$ABCD$ 為長方形，E、F、G、H 分別為 AB、BC、CD、DA 上的點，且 $EG=3$，$FH=4$。求長方形周界的最大可能值。**

In the figure, $ABCD$ is a rectangle. E, F, G, H are points on AB, BC, CD, DA respectively with $EG=3$, $FH=4$. Find the largest possible perimeter of the rectangle.



第 16 至第 20 題，每題 8 分。

Questions 16 to 20 each carries 8 marks.

16. **一所飲品公司正推行一項以印花換領飲品的計劃。此飲品乃非賣品，但每 11 個印花可換領 1 瓶飲品。而且，每瓶飲品上皆有 1 個印花以作日後換領之用。如果某人希望換領 20020302 瓶飲品，那麼他最少需要多少個印花？**

A beverage company is promoting a scheme of drink redemption by stamps. The drinks are not for sale, but every 11 stamps can redeem a bottle of drink. Moreover, there is a stamp on each bottle of drink for further redemption. If someone wants to redeem 20020302 bottles of drink, how many stamps does he need?

17. **張小姐和李小姐在一個圓形公路上踏單車，二人出發的地點分別在這個圓形公路的一條直徑之兩端。她們同時出發，出發後她們的速度一直保持不變。如果她們都是以逆時針的方向行駛，她們會在 80 分鐘後相遇；如果李小姐以逆時針的方向行駛，張小姐以順時針的方向行駛，她們會在 10 分鐘後相遇。已知李小姐和張小姐的速度分別為 36 km/h 和 x km/h，而且李小姐比張小姐快，求 x。**

Miss Cheung and Miss Lee are riding bicycles around a circular track. Their starting points are on two endpoints of a diameter of the circular track. They start cycling at the same time. After they have started cycling, their speeds keep unchanged. If both of them ride anti-clockwise, they will meet after 80 minutes. If Miss Lee rides anti-clockwise while Miss Cheung rides clockwise, they will meet after 10 minutes. Given that the speeds of Miss Lee and Miss Cheung are 36 km/h and x km/h respectively, and that Miss Lee rides faster than Miss Cheung find x.

18. **現有 10 塊等邊三角形卡片和 n 塊正五邊形卡片（$n > 0$）。每塊三角形和五邊形的卡片的邊長皆相等。已知全部卡片可以構成一個多面體（這些卡片全都用作這個多面體的面）。求 n 的最小可能值。**

There are 10 pieces of cardboard in the shape of an equilateral triangle and n pieces of cardboard in the shape of a regular pentagon ($n > 0$). The lengths of the sides of all the triangles and pentagons are the same. It is known that using all these pieces, a polyhedron can be constructed (all pieces of cardboard are used as faces of the polyhedron). Find the smallest possible value of n.

19. **某個會有** 2002 **名會員。除該會的創會會員外，每人均需得到剛好一名已入會的會員推薦方能入會。每名會員獲發一張會員證。基於行政上的理由，每名非創會會員的會員證均需與推薦人的會員證同色，而不同創會會員之會員證的顏色不同。若一名會員曾經推薦新會員入會，則被推薦的人數必定為** 12**、**30 **或** 42**。此外，沒有會員於入會後退會。問該會的會員證至少有多少種不同的顏色？**

A club has 2002 members. Except for its founders, each person must be nominated by exactly one existing member before he can join the club. Every member is issued a membership card. For administrative purpose, the membership card of each non-founding member must be of the same colour as that of one who nominated him, and the membership cards of different founders are of different colours. If a member of the club has ever nominated new members, the number of nominees must be 12, 30 or 42. Also, no member ever drops out after joining the club. What is the minimum number of different colours of the membership cards?

20. **有一個正方體，現把每個頂點塗上紅色或藍色，使得紅色頂點比藍色頂點多。問有多少種不同的塗色方法？（若兩個正方體不論如何旋轉均無法達致對應頂點顏色相同，則我們說這兩個正方體的塗色方法不同，否則我們說這兩個正方體的塗色方法相同。）**

There is a cube and each of its vertices is coloured red or blue, such that there are more red vertices than blue vertices. How many different ways of colourings are there? (Two cubes are said to have different colourings if it is impossible to make the colours of their corresponding vertices the same upon any rotation. Otherwise we say that they have the same colouring.)

全卷完

END OF PAPER

個人賽（中一組）答案

Individual Event (Secondary 1) Answers

1. 2
2. 2
3. 500.5
4. 216
5. 22
6. 2001
7. 5
8. $\frac{3}{11}$
9. 800
10. 50
11. 16
12. 37
13. 572
14. 1999
15. 14
16. 200203021
17. 28
18. 2
19. 4
20. 8