香港培正中學第一屆數學邀請賽

Pui Ching Middle School 1st Invitational Mathematics Competition

團體賽（初級組）

Group Event (Junior Section)

時限：45 分鐘

Time allowed: 45 minutes

參賽者須知：

Instructions to Contestants:

1. **本卷共設** 20 **題，總分為** 100 **分**。

   There are 20 questions in this paper and the total score is 100.

2. **除特別指明外，本卷內的所有數均為十進制**。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. **除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值**。

   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. **所有答案填在答題紙指定的空位上。毋須呈交計算步驟**。

   Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. **不得使用計算機**。

   The use of calculators is not allowed.

6. **本卷的附圖不一定依比例繪成**。

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

Questions 1 to 5 each carries 2 marks.

1. **求最小的正整數 n，使得 $80-n$ 和 $80+n$ 均為質數。**

   Find the smallest positive integer n for which both $80-n$ and $80+n$ are prime.

2. **若某年 5 月 28 日為星期四，且同年六月 k 日為星期一，求 k 的最大可能值。**

   If 28th of May of a certain year is Thursday and the k-th day of June in the same year is Monday, find the largest possible value of k.

3. **若九位數 $2002k0302$ 可被 9 整除，求 k 所有可能值之和。**

   If the 9-digit number $2002k0302$ is divisible by 9, find the sum of all possible values of k.

4. **某年四月的平均氣溫為 21 度。若該年四月一日至十日的平均氣溫為 17 度，則該年四月十一日至三十日的平均氣溫是多少度？**

   The average temperature in April of a certain year was 21 degrees. If the average temperature from 1st to 10th April that year was 17 degrees, what was the average temperature from 11th to 30th April that year in degrees?

5. **現有 10 種不同顏色的襪子，每種顏色有 10 隻。問至少要抽出多少隻襪子，才能確保三個人可以各自從中取得兩隻同色的襪子？**

   There are socks of 10 different colours and 10 socks of each colour. What is the smallest number of socks that must be drawn to ensure that three people can each choose two socks of the same colour from those drawn?

第 6 至第 10 題，每題 4 分。

Questions 6 to 10 each carries 4 marks.

6. **一間文具店有鉛筆出售：每支鉛筆售 1 元，5 支裝售 4 元，17 支裝售 12 元。如果李先生需要 2002 支鉛筆，並付了 n 元買鉛筆，那麼 n 的最小值是甚麼？（注意他買的鉛筆可能比他需要的多。）**

   A shop sells pencils. Each pencil costs 1 dollar, a bulk package of 5 pencils costs 4 dollars and a bulk package of 17 pencils costs 12 dollars. If Mr Lee needs 2002 pencils and he pays n dollars to buy pencils, what is the smallest value of n? (Note that he may buy more pencils than he needs.)

7. **三根鐵棒 A、B 和 C 垂直地插進盛了水的水池中。原先 A、B、C 露出水面部份的長度之比為 $1:2:4$。下雨後，水深是原來的兩倍。那時，A、B、C 露出水面部份的長度之比為 $1:3:7$。若 A 的長度為 20 米，且 C 的長度為 x 米，求 x。**

   Three iron bars, A, B and C, were inserted vertically into a water tank with water. Initially, the ratio of the lengths of A, B and C above water was $1:2:4$. After raining, the depth of water was doubled. At that time, the ratio of the lengths of A, B and C above water became $1:3:7$. If the length of A is 20 metres and the length of C is x metres, find x.

8. **若把 1 到 2002 之間所有 3 的倍數加起來，然後除以 100，餘數是多少？**

   If we add up all the multiples of 3 between 1 and 2002 and then divide by 100, what will be the remainder?

9. **對所有正整數 n，定義 $1+2+3+...+n$ 為第 n 個三角形數。有多少個三角形數不是合成數？**

   For all natural numbers n, define the n-th triangular number by $1+2+3+...+n$. How many triangular numbers are not composite?

10. **某次數學測驗共有 20 題。每題答對可得 5 分，不答得 0 分，答錯扣 2 分。若小麗在測驗中的得分為質數，問她最多答對了幾題？**

    In a mathematics test there were 20 questions. 5 marks were awarded for each correct answer, 0 mark would be given if a question was left unanswered, and 2 marks were deducted for each wrong answer. If the score of Lily in the test is a prime number, what is the largest number of questions she answered correctly?

第 11 至第 15 題，每題 6 分。

Questions 11 to 15 each carry 6 marks.

11. **已知對於正整數 n，$1^2+2^2+\cdots+n^2=\dfrac{n(n+1)(2n+1)}{6}$。**

    求 $1\times3+2\times4+3\times5+\cdots+200\times202$ 的值。

    Given that for positive integers n, $1^2+2^2+\cdots+n^2=\dfrac{n(n+1)(2n+1)}{6}$.

    Find the value of $1\times3+2\times4+3\times5+\cdots+200\times202$.

12. **某國家只有三元和五元兩種硬幣。對於正整數 n，若我們能用這兩種硬幣湊成 n 元，則我們說 n 是「好數」。例如，4 個三元硬幣和 3 個五元硬幣可湊成 27 元，故 27 是「好數」。有多少個正整數不是「好數」？**

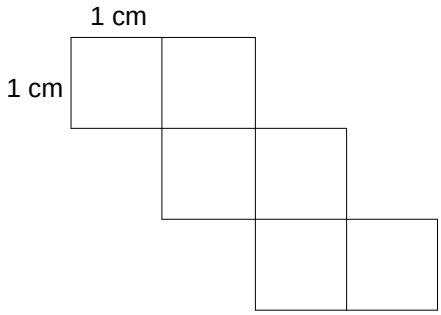
    In a country, there are only two types of coins, of $3 and $5 denominations. A positive integer n is said to be 'good' if we can make up an amount of n dollars using these two types of coins. For example, 27 is 'good' since four $3 coins and three $5 coins together make up an amount of $27. How many positive integers are not 'good'?

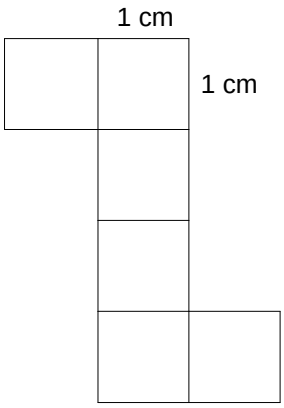
13. **某人到超級市場購物，付款 500 元。收銀員找贖時誤把 100 元紙幣當成 50 元，50 元紙幣當成 20 元，20 元紙幣當成 100 元。已知收銀員共找回紙幣六張，且超級市場因她出錯而損失了 110 元。問該人所選購的貨品總值多少元？**

    A man went shopping in a supermarket and paid $500. When returning the change, the cashier misregarded the $100 notes as $50 notes, $50 notes as $20 notes and $20 notes as $100 notes. Given that 6 notes have been returned as change and the supermarket lost $110 as a result of the cashier's fault, how many dollars were the items bought by the man worth?

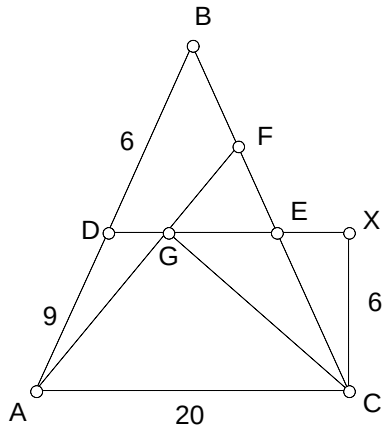
14. **附圖是一些可以摺成一個邊長為 1 厘米的立方體的紙樣。現要摺一個長 3 厘米，闊 4 厘米，高 5 厘米的長方體。如果所需要的紙樣的周界最短是 n 厘米，求 n。**

The figures below show some nets which can be folded to form a cube with side length 1 cm. Now a rectangular block which is 3 cm long, 4 cm wide and 5 cm high is to be folded. If the minimum perimeter of the net needed is n cm, find n.





15. **我們以 $[XYZ]$ 來表示 ΔXYZ 的面積。圖中，ABC 是三角形，D、E 分別為 AB 及 BC 上的點，使得 $DE // AC$、$BD = 6$ 及 $AD = 9$。若 X 在 DE 的延線上，使得 CX 與 DE 垂直及 $CX = 6$，F 是線段 BE 上任意一點，且 G 是 AF 與 DE 的交點，求 $[ADG] + [GEC]$ 的值。**



We denote the area of ΔXYZ by $[XYZ]$. In the figure, ABC is a triangle, D and E are points on AB and BC respectively such that $DE // AC$, $BD = 6$ and $AD = 9$. If X is a point on DE produced such that CX is perpendicular to DE and $CX = 6$, F is an arbitrary point on line segment BE and G is the intersection point of AF and DE, find the value of $[ADG] + [GEC]$.

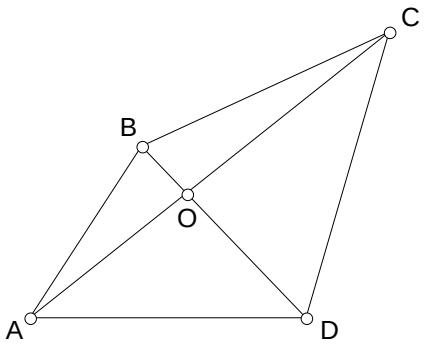
第 16 至第 20 題，每題 8 分。

Questions 16 to 20 each carries 8 marks.

16. **一張尺寸為 11×11 的正方形白色氈子被分成 121 個尺寸為 1×1 的方格。氈子正中間的白色方格隨後被塗上黑色。那麼包括這個黑色方格的長方形（包括正方形）共有多少個？**

A square white mat of size 11×11 is divided into 121 squares of size 1×1. The square in the middle of the mat is then painted black. How many rectangles (INCLUDING squares) contain this black square?

17. **我們以 $[XYZ]$ 來表示 ΔXYZ 的面積。圖中，$ABCD$ 為四邊形，AC 交 BD 於 O。若 $AO=4$、$CO=5$、$DO=3$、$AD=6$，且 $BD=CD$，求 $\dfrac{[AOB]}{[COD]}$ 的值。**



We denote the area of ΔXYZ by $[XYZ]$. In the figure, $ABCD$ is a quadrilateral; AC and BD meet at O. If $AO = 4$, $CO = 5$, $DO = 3$, $AD = 6$ and $BD = CD$, find the value of $\dfrac{[AOB]}{[COD]}$.

18. **求最小的正整數 n，使得 $n+256$ 為 625 的倍數，且 $n+625$ 為 256 的倍數。**

Find the smallest positive integer n for which $n+256$ is a multiple of 625 and $n+625$ is a multiple of 256.

19. **有一個壞了大部份按鍵的計算機，只剩下三個數字鍵「0」、「6」和「7」和兩個功能鍵「+」（加號）和「=」（等號）可正常操作。現在它的螢幕上顯示著「0」。計算機的設計不容許連續兩次按功能鍵。如果我們希望通過運算令螢幕顯示「2002」，最少需要按數字鍵（不包括功能鍵）多少次？（註：輸入整數時每一個數字皆需按一次鍵，例如輸入「706」和「67077」時分別需要按 3 次和 5 次鍵。）**

There is a broken calculator with only three number keys '0', '6' and '7' and two function keys '+' (plus) and '=' (equals) still working well. '0' is shown on its screen now. The design of the calculator does not allow two consecutive presses of function keys. If we want to show '2002' on its screen through calculations, what is the least number of presses of number keys (NOT including function keys) we need? (Note: When inputting an integer, each digit needs a press to be input. For example, inputting '706' and '67077' need 3 and 5 presses of number keys respectively.)

20. **有一個五邊形 *ABCDE*。現把每個頂點塗上紅色、黃色、綠色或藍色，使得相鄰的頂點所塗上之顏色不同。問共有多少種不同的塗色方法？**

There is a pentagon *ABCDE*. Each vertex is to be painted red, yellow, green or blue in such a way that adjacent vertices are painted in different colours. How many different ways of colourings are there?

全卷完

END OF PAPER

團體賽（初級組）答案

Group Event (Junior Section) Answers

1. 9
2. 29
3. 9
4. 23
5. 15
6. 1415
7. 60
8. 34
9. 2
10. 17
11. 2726900
12. 4
13. 240
14. 50
15. 24
16. 1296
17. $\frac{2}{5}$
18. 159119
19. 9
20. 240