

香港培正中學第二屆數學邀請賽

Pui Ching Middle School 2nd Invitational Mathematics Competition

個人賽（中三組）

Individual Event (Secondary 3)

時限：1 小時 30 分

Time allowed: 1 hour 30 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 若 n 位數 $44\dots44$ 可被 9 整除，求 n 的最小可能值。

If the n -digit number $44\dots44$ is divisible by 9, find the smallest possible value of n .

2. 若把所有小於 100 的奇質數乘起來，所得結果的個位數字是甚麼？

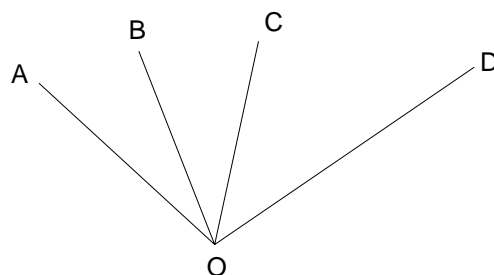
If we multiply together all odd primes less than 100, what will be the unit digit of the result?

3. 某 n 邊形的其中三隻內角分別為 144° 、 244° 及 344° 。求 n 的最小可能值。

Three interior angles of an n -sided polygon are 144° , 244° and 344° . Find the smallest possible value of n .

4. 圖中， $\angle AOD = 120^\circ$ ， $3\angle AOB = \angle BOD$ ，且 $\angle AOC = 2\angle COD$ 。求 $\angle BOC$ 。

In the figure, $\angle AOD = 120^\circ$, $3\angle AOB = \angle BOD$ and $\angle AOC = 2\angle COD$. Find $\angle BOC$.



第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 小強寫下了所有不超過 2003 而至少有一個「2」字的正整數，即 2, 12, 20, 21, ..., 2002, 2003。小強共寫下了多少個整數？

Peter wrote down all positive integers not exceeding 2003 which have at least one '2' in their digits, i.e. 2, 12, 20, 21, ..., 2002, 2003. How many integers did Peter write down?

6. 一位老師把 2003 名學生分成若干組，並記下了每組的人數。他最多會記下多少個不同的正整數？

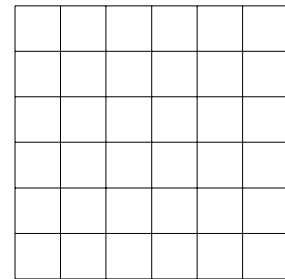
A teacher divides 2003 students into a certain number of groups, and the number of students in each group is recorded. At most how many different positive integers can be recorded?

7. 求 $\log_{10} \log_{10} \underbrace{\sqrt[10]{\sqrt[10]{\dots\sqrt[10]{10}}}}_{2003 \text{ 個根號}}$ 的值。

Find the value of $\log_{10} \log_{10} \underbrace{\sqrt[10]{\sqrt[10]{\dots\sqrt[10]{10}}}}_{2003 \text{ radicals}}$.

8. 在 6×6 的方格棋盤中放六顆相同的棋子，使得每橫行和每直行均只有一顆棋子，問共有多少種放法？

How many ways are there to put six identical pieces of chess on a 6×6 square chessboard such that each row and each column contains only one piece of chess?



第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 求一個四位的平方數，使得當每位數字均加上 3 時，所得的四位數仍是平方數。

Find a four-digit square number such that when each digit is increased by 3, the resulting four-digit number is still a square number.

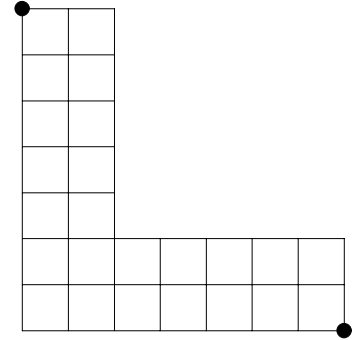
10. 一張尺寸為 10×10 的氈子被分成 100 個尺寸為 1×1 的方格。氈子左上角和右下角的方格隨後被割去。那麼氈子上有多少個長方形（包括正方形）？

A mat of size 10×10 is divided into 100 squares of size 1×1 . The squares at the top left and bottom right corners are then removed. How many rectangles (including squares) can be found on the mat?

11. 在直角坐標平面上， x 坐標和 y 坐標均為整數的點稱為「格點」。若以原點為中心，半徑為 $\sqrt{1000}$ 畫一個圓形，這個圓形的圓周會穿過多少個格點？

On the rectangular coordinate plane, points whose x -coordinates and y -coordinates are both integers are called 'lattice points'. If a circle is drawn with the origin as centre and $\sqrt{1000}$ as radius, how many lattice points will the circumference of this circle pass through?

12. 如圖所示，我們從一個 7×7 方格陣的右上角移去一個 5×5 的方格陣，從而得到一個 L 形方格陣。若要從這 L 形方格陣的左上角走至右下角，且每步只能沿格線向右或向下走，共有多少種不同的走法？



Given a 7×7 grid, we remove the 5×5 grid at the upper right hand corner to obtain an L-shaped grid as shown. We want to travel from the upper left hand corner to the lower right hand corner of this L-shaped grid, but in each step we can only move rightward or downward along the gridlines. How many different paths are there?

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 若 n 為正整數，且 $n^2 = 11112222^2 + 3333^2 + 3334^2$ ，求 n 。

If n is a positive integer and $n^2 = 11112222^2 + 3333^2 + 3334^2$, find n .

14. 15 名學生（中一級至中五級各三名）參加一個工作坊。他們分成三人一組，並規定每組中任何兩位組員最多只可以相差一級。共有多少種不同的方法把學生分組？

15 students, three from each of secondary 1 to secondary 5, take part in a workshop. They are to be divided into groups of three, and it is required that any two members in the same group differ by at most one form. In how many different ways can we divide the students into groups?

15. 潘先生對小敏和小賢說：「我想了兩個正整數 a 和 b ，其中 $a > b$ 。」然後他秘密地將兩數之差 ($a - b$) 告訴小敏，及將兩數之積 (ab) 告訴小賢。以下是他們之後的對話。

潘先生問小敏：「你知道 a 和 b 是甚麼嗎？」

小敏說：「不知道。」

然後潘先生問小賢：「你知道 a 和 b 是甚麼嗎？」

小賢說：「我知道啊。它們是 _____」

潘先生立即打斷了小賢的說話，並再問小敏：「現在你知道 a 和 b 是甚麼嗎？」

這時小敏回答：「聽過你和小賢剛才的對話後，我知道它們是甚麼了。它們的和 $a + b$ 大於 90，小於 100。潘先生，對嗎？」

潘先生說：「是啊，你們真聰明呢。」

假設小賢和小敏都是誠實和聰明的（即是說只要當答案可以確定時，他們一定知道答案），求 ab 。

Mr Poon told Dora and Ken: 'I thought of two positive integers a and b , where $a > b$.' He then secretly told Dora the difference of the two numbers (i.e. $a - b$) and Ken the product of the two numbers (i.e. ab). Their subsequent conversation is recorded below.

Mr Poon asked Dora, 'Do you know what a and b are?'

'No.' Dora answered.

Then Mr Poon asked Ken, 'Do you know what a and b are?'

'Yes, they are ...' Ken said.

Mr Poon interrupted Ken immediately and asked Dora again, 'Do you know what a and b are now?'

At that time, Dora said, 'Now I know what they are after listening to the conversation between Ken and you. Their sum (i.e. $a + b$) is greater than 90 but less than 100. Am I right, Mr Poon?'

'Yes. You are so clever!' Mr Poon said.

Assuming that Ken and Dora are honest and intelligent (it means that whenever the answer can be confirmed, they must know the answer), find ab .

16. 2003 位小朋友被編成 1 至 2003 號。他們每人最初有 20 顆糖果，然後按以下方法派給或拿走糖果：先派 3 顆糖果給每位編號是 1 的倍數的小朋友，再從每位編號是 2 的倍數的小朋友手上取走 1 顆糖果，然後派 3 顆糖果給每位編號是 3 的倍數的小朋友，再從每位編號是 4 的倍數的小朋友手上取走 1 顆糖果，如此類推，直至最後派 3 顆糖果給每位編號是 2003 的倍數的小朋友。最後有多少個小朋友比最初多了糖果（即有 21 顆糖果或以上）？

2003 children are labelled 1 to 2003. Each of them has 20 candies at the beginning. Candies are then given to or taken away from them as follows. 3 candies are given to each child with a label which is a multiple of 1, then 1 candy is taken away from each child with a label which is a multiple of 2, then 3 candies are given to each child with a label which is a multiple of 3, then 1 candy is taken away from each child with a label which is a multiple of 4, and so on, till finally 3 candies are given to each child with a label which is a multiple of 2003. How many children can get extra candies (i.e. have 21 candies or more) in the end?

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 小明和小芳玩一個遊戲。首先小明在一個 13×13 的方格表中放了 5 枚硬幣，使得這 5 枚硬幣放在 5 個位於同一橫行或同一直行的相鄰方格中。然後，小芳開始猜這些硬幣的位置。她每次會選一個方格，如果她選中了一個有硬幣的方格，她便獲勝。小芳最少要猜多少次才可以確保獲勝？

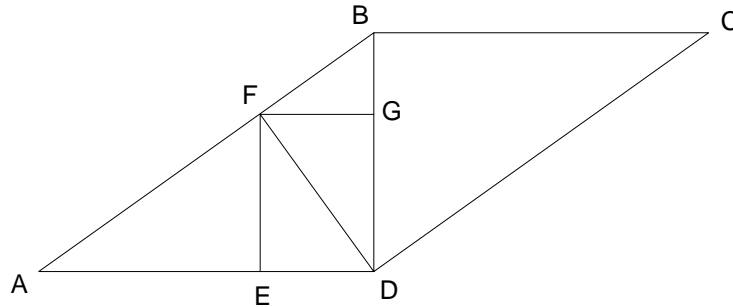
Alan and Betty play a game. First, Alan puts 5 coins on a 13×13 square grid such that the 5 coins are in 5 neighbouring cells lined up in a row or a column. Then Betty starts to guess where the coins are. Every time when she guesses, she chooses a cell. If the cell is occupied by a coin, she wins. What is the minimum number of guesses she has to make to ensure that she wins?

18. 設 $ABCDE$ 為一個以菱形為底的錐體，其中 E 是錐體的頂點。已知菱形 $ABCD$ 的邊長是為 11， $AE = CE = 12$ ，且 $BE = DE = 11$ 。求錐體的體積。

Let $ABCDE$ be a pyramid with a rhombus as base and E as vertex. Given that the side length of the rhombus $ABCD$ is 11, $AE = CE = 12$ and $BE = DE = 11$. Find the volume of the pyramid.

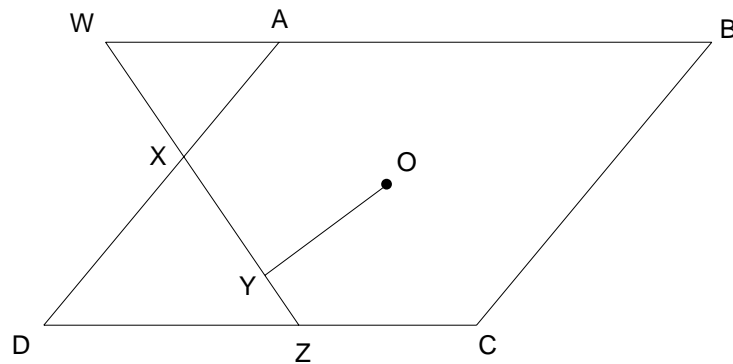
19. 圖中， $ABCD$ 為平行四邊形。 E 是 AD 上的一點， F 是 AB 上的一點， G 是 BD 上的一點，使得 $DEFG$ 為長方形。若 $BG = 1$ ， $CD = 8$ ，且 $FD \perp CD$ ，求 AE 的長度。

In the figure, $ABCD$ is a parallelogram. E is a point on AD , F is a point on AB and G is a point on BD such that $DEFG$ is a rectangle. If $BG = 1$, $CD = 8$ and $FD \perp CD$, find the length of AE .



20. 如圖所示， $ABCD$ 為菱形， X 及 Z 分別是 AD 及 DC 上的點。 O 是菱形內的一點， Y 為 O 到 XZ 的垂足，且 O 與 AB 、 BC 、 CD 和 DA 的距離均等於 OY 。若 ZX 與 BA 延長後交於 W ，且 $WX = 5$ ， $XY = 3$ 及 $YZ = 2$ ，求 $ABCD$ 的面積。

In the figure, $ABCD$ is a rhombus. X and Z are points on AD and DC respectively. O is a point inside the rhombus, Y is the foot of the perpendicular from O to XZ and the distances from O to AB , BC , CD and DA are all equal to OY . If ZX and BA are produced to meet at W , $WX = 5$, $XY = 3$ and $YZ = 2$, find the area of $ABCD$.



全卷完

END OF PAPER

個人賽 (中三組) 答案

Individual Event (Secondary 3) Answers

1. 9

13. 11112223

2. 5

14. 280

3. 7

15. 97

4. 50°

16. 1753

5. 546

17. 33

6. 62

18. 336

7. -2003

19. $3\sqrt{3}$

8. 720

20. 80

9. 1156

10. 2826

11. 16

12. 491