

香港培正中學第二屆數學邀請賽

Pui Ching Middle School 2nd Invitational Mathematics Competition

團體賽（初級組）

Group Event (Junior Section)

時限：45 分鐘

Time allowed: 45 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 某長方體的長是闊的兩倍，高是闊的三倍。若它的體積為 48，它的總表面積是多少？

A cuboid has length twice its width and height three times its width. If its volume is 48, what is its total surface area?

2. 某正方形的周界為 k 單位，面積為 k 平方單位。求 k 。

A square has perimeter k units and area k square units. Find k .

3. 求 $\underbrace{1+1-1\times 1\div 1+1-1\times 1\div 1+\cdots}_{1234 \text{ 個「1」}}$ 的值。

Find the value of $\underbrace{1+1-1\times 1\div 1+1-1\times 1\div 1+\cdots}_{1234 \text{ '1's}}$.

4. 求具以下性質的 2003 位正整數的數目：除首兩位數字外，其餘所有數字均等於之前兩位數字的差（例如：374312110...）。

How many 2003-digit positive integers have the property that except for the first two digits, each digit is equal to the difference between its two preceding digits (e.g. 374312110...)?

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 設 $[x]$ 代表小於或等於 x 的最大整數，例如： $[2.34] = 2$ ， $[-2.5] = -3$ ， $[7] = 7$ 等。有多少個正整數 n 可使得 $\left[\frac{n^2}{3} \right]$ 為質數？

Let $[x]$ denote the greatest integer less than or equal to x . For example, $[2.34] = 2$, $[-2.5] = -3$, $[7] = 7$ and so on. How many positive integers n are there such that $\left[\frac{n^2}{3} \right]$ is prime?

6. 某村莊獲派一些食水。已知每人每天喝水的量是一樣的。若他們每人每天少喝 1 升水，則食水可以維持多 15 天。若他們每人每天少喝 1.5 升水，則食水可以維持多 30 天。現在食水可以維持多少天？

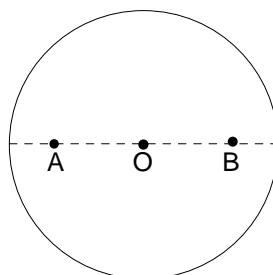
Some water is distributed to a village. It is known that everyone drinks the same amount of water every day. If each villager drinks 1 litre less per day, the water distributed can last for 15 more days. If each of them drinks 1.5 litres less per day, the water distributed can last for 30 more days. How many days can the water distributed last?

7. 我們以 \overline{ABC} 代表一個百位為 A 、十位為 B 及個位為 C 的三位數。小明約簡分數時，誤將一個分數 $\frac{\overline{ABC}}{\overline{BCD}}$ 看成 $\frac{A \times B \times C}{B \times C \times D}$ 。因此他將之約簡成 $\frac{A}{D}$ 。但他得出的答案和真正的答案卻巧合地相同。已知 $A = 7$ 和 $\overline{ABC} \neq 777$ ，求 \overline{ABC} 。

We denote by \overline{ABC} a three-digit number with hundreds digit A , tens digit B and unit digit C . When Peter reduces fractions, he wrongly regards a fraction $\frac{\overline{ABC}}{\overline{BCD}}$ as $\frac{A \times B \times C}{B \times C \times D}$. Therefore, he reduces it to $\frac{A}{D}$. However, he gets the same answer as the correct one coincidentally. Given that $A = 7$ and $\overline{ABC} \neq 777$, find \overline{ABC} .

8. 在一個圓形的湖上有兩個燈塔 A 和 B 。湖的中心 O 位於它們的中點，且兩個燈塔的距離是湖的直徑的三分之二。每座燈塔的服務範圍定義為湖面上距離此燈塔比距離其他燈塔都近的部分。現在政府計劃在湖上設立第三座燈塔 C ，使得三個燈塔服務範圍的面積相同。 C 的位置有多少個不同的選擇？

In a circular lake there are two lighthouses A and B . The centre of the lake O lies at their midpoint, and the distance between the lighthouses is two-thirds of the diameter of the lake. The serving area of each lighthouse is defined to be the parts of the lake surface such that it is closest to that lighthouse when compared with any other lighthouses on the lake. Now the government plans to set up a third lighthouse C on the lake such that all three lighthouses have the same serving area. How many choices of the position of C does the government have?



第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 已知 $\frac{1}{13} = 0.07692\dot{3}$ 。若把 $\frac{23}{130}$ 以小數表示，小數點後首 2003 位數字之和是多少？

Given $\frac{1}{13} = 0.07692\dot{3}$. If $\frac{23}{130}$ is written as a decimal, what is the sum of the first 2003 digits after the decimal point?

10. 一張尺寸為 16×16 的紙被分成 256 個尺寸為 1×1 的小正方形。小怡沿著這些小正方形的邊界，將紙切割成若干面積互不相同的長方形（包括正方形）小塊。她最多可以得到多少小塊？

A piece of paper of size 16×16 is divided into 256 small squares of size 1×1 . Along the sides of the small squares, Emma cuts the paper into rectangular (including square) pieces having pairwise different areas. At most how many pieces can she get?

11. 某數列的首六項為 2000、2003、2002、2000、2000、1999。由第七項起，每項均等於 $a + 3b + 9c + 27d + 81e + 243f$ 除以 6 時的餘數，這裡 a 、 b 、 c 、 d 、 e 和 f 分別是之前的六項（ a 是六項前的數， b 是五項前的數，如此類推）。此數列中連續六項的和最小是多少？

The first six terms of a sequence are 2000, 2003, 2002, 2000, 2000 and 1999. Starting from the seventh term, each term is equal to the remainder when $a + 3b + 9c + 27d + 81e + 243f$ is divided by 6. Here a , b , c , d , e and f denote the previous six terms (a is the number six terms before, b is five terms before, and so on). What is the minimum sum of six consecutive terms in this sequence?

12. 小雄的老師給了小雄 5 個正整數。小雄每次將兩個乘起來，得出了 10 個不同的乘積，可是小雄只記得其中 5 個積是 63、80、112、140 和 168。求原來的 5 個正整數之和。

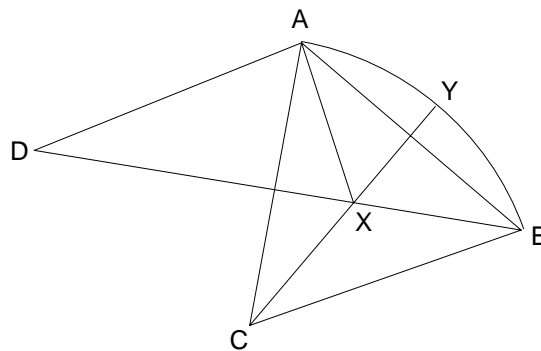
Peter's teacher gives him 5 positive integers. Peter multiplies two of them each time to get 10 different products. However, Peter only remembers 5 of the products, namely, 63, 80, 112, 140 and 168. Find the sum of the 5 original numbers.

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 圖中， B 、 C 及 D 為一個以 A 為圓心的圓上的三點（此圓沒有畫出來）。 BXD 及 CXY 為直線， $BX = XC$ ，且 AYB 是一個以 C 為圓心的弧。若 $AD = XD$ ，求 $\frac{\text{扇形 } ACY \text{ 的面積}}{\text{扇形 } BCY \text{ 的面積}}$ 。

In the figure, B , C , D are points on a circle centred at A (the circle is not shown here). BXD and CXY are straight lines such that $BX = XC$, and AYB is an arc centred at C . If $AD = XD$, find $\frac{\text{area of sector } ACY}{\text{area of sector } BCY}$.



14. 若正整數 n 符合以下兩個條件，則稱為「好數」：

- (1) n 的最後一位數字不是 0；
- (2) 若我們把 n 的各位數字左右倒轉，則所得的數為 n 的倍數。

在 $10 < n < 10000$ 範圍內，共有多少個「好數」？

A positive integer n is said to be 'good' if

- (1) its last digit is not zero; and
- (2) when we reverse the digits of n , the number obtained is a multiple of n .

How many 'good' numbers are there in the range $10 < n < 10000$?

15. 求一個小於 800 的正整數 n ，使得 n 剛好有 20 個正因數，且 n 不能被 16 整除。

Find a positive integer n less than 800 such that n has exactly 20 positive factors and is not divisible by 16.

16. 某班共有 45 名學生，他們的班號分別為 1、2、...、45。現要選 22 名學生參加一項活動，當中被選出的任何兩名學生的班號必須最少相差 2。共有多少種不同的方法選擇學生？

In a class there are 45 students. Their class numbers are 1, 2, ..., 45. Now 22 students are to be chosen to attend an activity, and the class numbers of any two chosen students must differ by at least 2. How many ways are there to choose the students?

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 某次測驗共有 100 題選擇題，每題答對可得 5 分，答錯扣 2 分，不答得 0 分。合格分數為 100。

如果一名學生回答 20 題，他必須全部答對方能合格。如果他多答一題，也必須同時答對該題才能合格（因為若他答錯了該題他便只得 98 分）。因此，若以合格為目標，則與其答 21 題，不如只答 20 題。我們說 21 是「壞數」。一般來說，若回答 n 題比回答 $n-1$ 題需要多答對一題才能合格，則我們說 n 是「壞數」。（因此求合格而聰明的學生知道，即使有信心答對 n 題，亦只應回答其中 $n-1$ 題。）

在 $21 \leq n \leq 100$ 範圍內，有多少個整數 n 是「壞數」？

In a test there are 100 multiple choice questions. For each question, 5 marks will be awarded for a correct answer, 2 marks will be deducted for a wrong answer, and 0 mark will be given if it is left unanswered. The passing mark is 100.

If a student answers 20 questions, he must get all 20 correct in order to pass. If he answers one more question, he must also get that question correct in order to pass (because if he gets it wrong he will only get 98 marks). So in order to pass, it will be better to answer only 20 rather than 21 questions. We say that 21 is a 'bad' number. In general, we say that n is 'bad' if answering n questions requires one more correct answer to pass than answering $n-1$ questions. (And those who are intelligent enough and aim to pass will only answer $n-1$ questions even if they are confident with n questions.)

How many 'bad' integers n are there in the range $21 \leq n \leq 100$?

團體賽 (初級組) 答案

Group Event (Junior Section) Answers

1. 88

13. 2

2. 16

14. 191

3. 2

15. 648

4. 90

16. 276

5. 2

17. 23

6. 30

18. 6

7. 742

19. 128

8. 2

20. 53

9. 9016

10. 21

11. 10

12. 56