

香港培正中學第三屆數學邀請賽
Pui Ching Middle School 3rd Invitational Mathematics Competition

團體賽（初級組）
Group Event (Junior Section)

時限：45 分鐘

Time allowed: 45 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設甲、乙兩部分，總分為 100 分。

This paper is divided into Section A and Section B. The total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.

No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

甲部 (60 分)

Section A (60 marks)

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

第 5 至第 8 題，每題 5 分。

Questions 5 to 8 each carries 5 marks.

第 9 至第 12 題，每題 7 分。

Questions 9 to 12 each carries 7 marks.

1. 某數學邀請賽的試卷中，每題皆佔 5 分或 7 分，滿分為 100 分。全卷最少有多少題？

In a certain invitational mathematics competition, each question carries 5 or 7 marks and the full score of the paper is 100. What is the minimum number of questions?

2. 有多少個三位正整數的數字之和可被 3 整除？

How many three-digit positive integers have their sum of digits divisible by 3?

3. 母親節是每年五月第二個星期日，父親節則是每年六月第三個星期日。若某年的父親節在母親節後 n 天，求 n 的所有可能值之和。

The Mother's Day is the second Sunday in May while the Father's Day is the third Sunday in June. In a certain year, the Father's Day comes n days after the Mother's Day. Find the sum of all possible values of n .

4. 設 a 、 b 為正整數， p 為質數。若 $51a + 17b = p^2$ ，求 a 的所有可能值之和。

Let a , b be positive integers and p be a prime number. If $51a + 17b = p^2$, find the sum of all possible values of a .

5. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 ABABA 所代表的五位數。

In the calculation shown, each alphabet represents a different digit from 0 to 9. Find the five-digit number represented by ABABA.

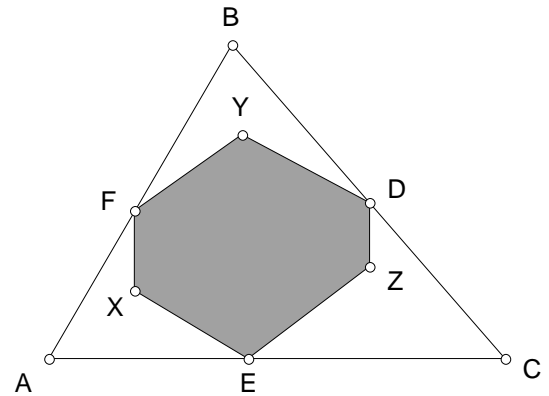
$$\begin{array}{r} \text{A B A B A} \\ \times \quad \text{A B A B A} \\ \hline \text{C C D D C 8 D C C} \end{array}$$

6. 已知 a 、 b 和 c 是三個符合 $100 > a > b > c > 0$ 的整數。若 $(a - b)$ 、 $(a - c)$ 和 $(b - c)$ 都是質數，求 $(a - c)$ 的最大可能值。

It is known that a , b and c are three integers satisfying $100 > a > b > c > 0$. If $(a - b)$, $(a - c)$ and $(b - c)$ are prime numbers, find the greatest possible value of $(a - c)$.

7. ABC 是一個面積為 10 的銳角三角形， D 、 E 和 F 分別是 BC 、 AC 和 AB 的中點。如果 X 、 Y 和 Z 是 $\triangle ABC$ 內的點，使得 $EX \perp AB$ 、 $FX \perp AC$ 、 $DY \perp AB$ 、 $FY \perp BC$ 、 $EZ \perp BC$ 和 $DZ \perp AC$ ，求六邊形 $DZEXFY$ 的面積。

In an acute triangle ABC of area 10, D , E and F are mid-points of BC , AC and AB respectively. If X , Y and Z are points inside $\triangle ABC$ such that $EX \perp AB$, $FX \perp AC$, $DY \perp AB$, $FY \perp BC$, $EZ \perp BC$ and $DZ \perp AC$, find the area of the hexagon $DZEXFY$.



8. 一個長方體的水缸內放了一些水和垂直插了兩支蠟燭。它們的長度分別是 37 cm 和 49 cm。小雄將兩支蠟燭點著，並同時用一條水喉向水缸加水。當蠟燭的頂部觸及水面時，蠟燭便會熄掉。已知當兩支蠟燭皆熄掉時，它們的長度分別是 25 cm 和 28 cm。那麼，水缸內的水原來的高度是多少厘米？（假設兩支蠟燭燃燒的速度一樣，而且在燃燒過程中保持不變。另外，水缸加水的速度亦一直不變。）

In a rectangular water tank, some water was poured in and two candles were placed vertically inside. Their lengths were 37 cm and 49 cm respectively. Simon lit the candles and filled the tank with a pipe. When the top of the candle met the water surface, the candle was put out. Given that when both candles were put out, their lengths were 25 cm and 28 cm respectively, find the original height (in cm) of water in the tank. (Assume that the consumption rates of the candles were identical and remained unchanged throughout the process. In addition, the rate of water filling remained unchanged too.)

9. 小明、小雄和小文三人在一個長 630 米的圓形跑道上跑步。他們在 10:00 於同一點出發，小明向順時針方向跑，小雄和小文則向逆時針方向跑，他們的速度一直保持不變。他們還規定了當其中兩人相遇時，相遇的兩人須立即改變方向。已知小明和小雄在 10:18 首次相遇，小雄和小文在 10:30 首次相遇，小明和小文在 10:44 首次相遇。小雄每分鐘跑多少米？

Alex, Ben and Calvin ran on a circular path of length 630 metres. They started at the same point at 10:00. Alex ran in the clockwise direction while Ben and Calvin ran in the anti-clockwise direction. Their speeds remained unchanged in the course. They also agreed that when two of them meet, the two must change their directions immediately. It is known that Alex and Ben first met at 10:18; Ben and Calvin first met at 10:30; Alex and Calvin first met at 10:44. How many metres did Ben run per minute?

10. 小怡、小芬、小婷、小紅和小雅都做了一個測驗，滿分為 10 分，合格分數為 2 分，每人的分數都是整數。小燕在成績公佈後問及她們的成績。以下是她們的回答：

小怡：「我們當中沒有兩個人得到相同的分數。」

小芬：「我和小怡的總分和另外三人的總分相同。」

小婷：「我和小紅的總分比另外三人的總分少 3 分。」

小紅：「沒有人不合格。」

小雅：「若把我們五人的分數乘起來，答案的最後兩位是 50。」

後來小燕發現有一個人說了謊話，其餘四人都說了真話。求她們五人的總分。

Amy, Betty, Cindy, Dora and Emma sat for a test with full mark 10 and passing mark 2. The score of each person is an integer. Fiona asked for their results after the results were announced. The following is their replies:

Amy: 'No two of us have the same score.'

Betty: 'The total score of Amy and I is equal to that of the other 3 students.'

Cindy: 'The total score of Dora and I is less than that of the other 3 students by 3.'

Dora: 'No one fails.'

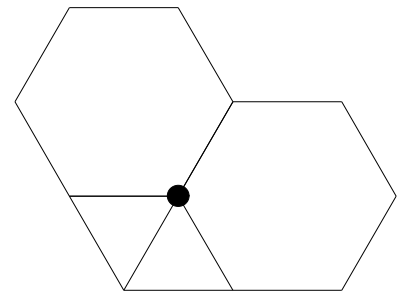
Emma: 'If our scores are multiplied, the last 2 digits of the result would be 50.'

Fiona found that one of them told a lie and the other four people told the truth. Find the sum of their five scores.

11. 某次測驗共有 100 題選擇題，每題答對可得 5 分，答錯扣 0.7 分，不答得 0 分。合格分數為 10。如果一名學生只回答 2 題，他必須同時答對 2 題方能合格。如果他多答一題，也必須同時答對該題才能合格（因為若他答錯了該題他便只得 9.3 分）。因此，若以合格為目標，則與其答 3 題，不如只答 2 題。我們說 3 是「壞數」。一般來說，對於不超過 100 的正整數 n ，若回答 n 題比回答 $n-1$ 題需要多答對一題才能合格，則我們說 n 是「壞數」。（因此求合格而聰明的學生知道，即使有信心答對 n 題，亦只應回答其中 $n-1$ 題。）若 n 不是「壞數」，便稱為「好數」。最多有多少個連續的「好數」？

In a test there are 100 multiple choice questions. For each question, 5 marks will be awarded for a correct answer, 0.7 mark will be deducted for a wrong answer, and 0 mark will be given if it is left unanswered. The passing mark is 10. If a student answers 2 questions, he must get both correct in order to pass. If he answers one more question, he must also get that question correct in order to pass (because if he gets it wrong he will only get 9.3 marks). So in order to pass, it will be better to answer only 2 rather than 3 questions. We say that 3 is a 'bad' number. In general, for a positive integer n not exceeding 100, we say that n is 'bad' if answering n questions requires one more correct answer to pass than answering $n-1$ questions. (And those who are intelligent enough and aim to pass will only answer $n-1$ questions even if they are confident with n questions.) If n is not 'bad', we say that it is 'good'. What is the greatest number of consecutive 'good' numbers?

12. 有些圖形由剛好四個邊長為 1 的正多邊形組成，其中各正多邊形有一個公共頂點，不互相重疊，而每個多邊形都跟剛好兩個其他的多邊形有一條公共邊。附圖所示的便是這種圖形的一個例子，它由兩個正六邊形和兩個正三角形組成，周界為 10。求這種圖形的周界的所有可能值之和。

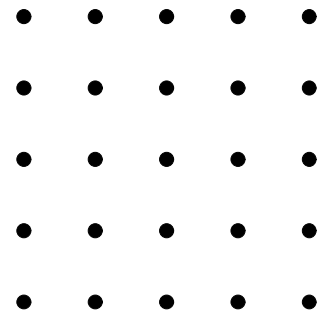


Some figures consist of exactly four regular polygons, each of side length 1, all of which share one common vertex and which do not overlap. Each polygon shares one common side with exactly two other polygons. For instance, the diagram shows one such figure, consisting of two regular hexagons and two equilateral triangles with perimeter 10. Find the sum of all possible perimeters of this type of figures.

乙部 (40 分)

Section B (40 marks)

13. 如圖所示，平面上有 25 點，它們以每行 5 點，每列 5 點的方式鋪成一個由 16 個全等的小正方形組成的大正方形。大正方形的面積為 16 平方單位。



As shown in the figure, 25 points are drawn on a plane, arranged in a way such that there are 5 points in each row and 5 points in each column, forming 16 congruent squares which comprise a large square. The area of the large square is 16 square units.

- (a) 若把每點塗上顏色，使得任何兩個相距 1 單位的點的顏色均不同，至少要多少種不同的顏色？ (3 分)

If we colour each point in a way such that any two points which are 1 unit apart have different colours, what is the minimum number of colours needed? (3 marks)

- (b) 若要在這 25 點中選其中 n 點 ($n \geq 3$) 成為一個凸多邊形的頂點，多邊形的面積有多少個不同的可能值？ (5 分)

If we choose n points ($n \geq 3$) among the 25 to be the vertices of a convex polygon, how many different possible areas of the polygon are there? (5 marks)

- (c) 若要在這 25 點中選其中 4 點，使得當中任何兩點之間的距離皆為整數，共有多少種選法？ (6 分)

If we choose 4 points among the 25 in a way such that the distance between any two of them is an integer, how many different choices are there? (6 marks)

- (d) 若要在這 25 點中選其中 m 點，使得當中任何兩點之間的距離互不相同，求 m 的最大可能值。 (6 分)

If we choose m points among the 25 in a way such that the distances between any two of them are pairwise distinct, find the greatest possible value of m . (6 marks)

14. 若某正整數可寫成 m^n 的形式，其中 m 、 n 均為大於 1 的整數，那麼該正整數便稱為「好數」。例如： $27 = 3^3$ ， $4096 = 64^2$ ，所以 27 和 4096 都是「好數」。

If a positive integer can be written in the form m^n , where m, n are integers greater than 1, then the integer is said to be 'good'. For example, since $27 = 3^3$ and $4096 = 64^2$, 27 and 4096 are 'good'.

- (a) 最小的「好數」是甚麼？ (2分)

What is the smallest 'good' number? (2 marks)

- (b) 小於 1000 的「好數」有多少個？ (4分)

How many 'good' numbers are smaller than 1000? (4 marks)

- (c) 有些「好數」有超過一種方法表達成 m^n 的形式（其中 m 、 n 均為大於 1 的整數），例如 4096 可寫成 64^2 ，亦可寫成 2^{12} 。那麼，有多少種不同的方法把 400^{400} 寫成以上的形式？ (7分)

Some 'good' numbers can be expressed in the form m^n (where m, n are integers greater than 1) in more than one way. For example, 4096 can be written in the form 64^2 as well as 2^{12} . In how many different ways can 400^{400} be written in the above form? (7 marks)

- (d) 設 a 、 b 、 c 為大於 1 的整數， $p = 2^a$ 、 $q = 2^b$ 、 $r = 2^c$ 。那麼， p 、 q 、 r 均為「好數」。若 $p^{2003} + q^{2004} = r^{2005}$ ，求 c 的最小可能值。 (7分)

Let a, b, c be integers greater than 1, $p = 2^a$, $q = 2^b$ and $r = 2^c$. Then p, q, r are 'good'. If $p^{2003} + q^{2004} = r^{2005}$, find the smallest possible value of c . (7 marks)

全卷完

END OF PAPER

團體賽 (初級組) 答案

Group Event (Junior Section) Answers

- | | | | |
|-----|-------|--------|---------|
| 1. | 16 | 13 (a) | 2 |
| 2. | 300 | 13 (b) | 32 |
| 3. | 77 | 13 (c) | 54 |
| 4. | 15 | 13 (d) | 5 |
| 5. | 21212 | 14 (a) | 4 |
| 6. | 73 | 14 (b) | 39 |
| 7. | 5 | 14 (c) | 17 |
| 8. | 21 | 14 (d) | 2006005 |
| 9. | 25 | | |
| 10. | 34 | | |
| 11. | 8 | | |
| 12. | 41 | | |